FVTD simulation of the acoustics of the Phonocamptic Cave in Noyon.

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ABSTRACT
Starting from new measurements of the acoustical pots and room geometry in the phonocamptic cave at the Cathedral of Noyon, a numerical study was undertaken to understand the acoustical effects at the boundaries, and to provide an auralization of the space. An implementation of the finite volume time domain (FVTD) method was used to model the cave, including fitting the impedance presented by the acoustical pots on certain boundaries. The individual impedances of the pots were estimated from impulse responses collected pot-by-pot and parameterized in terms of a Helmholtz resonator model. Then, using the electroacoustic analogy, the sum effect of the pots was modeled as an equivalent spatial distribution in the FVTD boundary conditions. Additionally, the space was discretized with an unstructured mesh in order to capture the complex geometry, minimize dispersion error, and to check the accuracy of the FVTD implementation. **Keywords**: acoustical pots, cave acoustics, simulation.

1. INTRODUCTION
The phonocamptic cave (or *caveau phonocamptique*) at the Cathedral of Noyon is a unique acoustic location, studied for its use of acoustical pots. While acoustical pots have been found in many ancient churches, the phonocamptic cave in Noyon has unique characteristics, as 64 pots are embedded in a room below the ground floor, under the cathedral's choir, as opposed to being placed in the high parts of the church's walls, which is the case for most churches.

Figure 1 – Acoustical pots in the West wall of the phonocamptic cave at Noyon

In many cases, it is believed that the introduction of pots into existing churches was an acoustical intervention intended to amplify the voices of singers, either for aesthetic or economic reasons [1]. This belief may have been based on the common observation that singing into a pot at a particular pitch, thus exciting its resonant frequency, can result in the pot "singing back" at the user. While we understand now that passive terminations cannot amplify sound in large spaces, the practical effect of these resonators in the phonocamptic cave is to increase resonant frequencies, so that the pots have a meaningful effect on the perceived acoustics of a space [2].

In this paper, we expand upon previous acoustical analyses of this space with new characterizations of individual pots and their collective boundary impedance, and integrate these findings into a numerical acoustical model intended to provide a basis for future studies of the space.

2. BACKGROUND

2.1 Previous Acoustical Studies
The cave has been the subject of previous studies that have aimed to characterize the space acoustically as well as ascertain the practical effect the pots induce on a group of singers [3].

This study collected measurements of the frequency-dependent reverberation time within the space, with pots open and closed, and at a variety of locations, serving as a ground truth for the overall

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acoustic behavior. This confirmed the impact of the pots within the cave itself, especially within the regions nearest particular groups of pots.

The study concluded with a performance within the space whose results suggested an improvement in the acoustics when transitioning from closed to open pots.

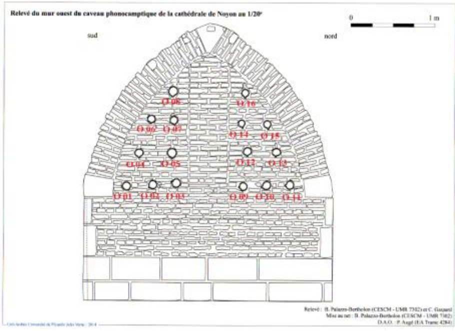

Figure 2 – Mapping of West wall with acoustical pots' positions

2.2 Numerical Models

In this study, we utilize two numerical models, the first being a characterization of the impedance of the acoustical pots (and other boundary surfaces of the cave), and the second a finite volume time domain (FVTD) model of acoustical wave motion within the space. Each of these individual models was the subject of a Master's internship undertaken by the primary authors [4, 5]. The goal of the study was to enable virtual auralization of the space and confirmation of the modal effects detected in the previous analysis of the space. Both models are well-known, but the nature of the space as well as the challenge of matching specific geometry and materials introduce new views onto each technique.

3. ACOUSTICAL POT MODEL

The first investigation was modeling the acoustical effects of the pots in the cave. In [1], the approximate resonant frequency of the pots was determined by clapping a hand over the mouth of each resonator and then singing into the pot to confirm the pitch. More recent measurements have enabled a more precise characterization of the individual pots in order to better model their combined effect [2, 3].

3.1 Measurements

Impulse responses of example pots on the North face of the cave were collected in order to understand the relationship of pot size to acoustical impedance using white noise as an excitation source. Additionally, the pot dimensions were measured with a laser rangefinder. The spectrum of the impulse response was used to find the width and peak of the resonant frequency, and the decay of the response was used to estimate the absorption of the pot [2, 3].

3.2 Acoustical Model

From these measurements, the impedance of a given pot could be calculated using a Helmholtz resonator as a model to fit, and a RLC circuit describing the damped oscillation of the air in and out of the opening could be defined using electroacoustical analogy.

For a given RLC circuit, the characteristic differential equation is:

$$L\ddot{u} + Ru + \frac{1}{C}u = 0 \qquad (1)$$

for a given inductance L, resistance R, and capacitance C. Similarly, the pots' resistance can be found using the system's damping ratio ζ, stiffness k and mass m.

Thus, with:

$$R = 2\zeta\sqrt{km}, \quad L = m, \quad C = \frac{1}{k} \qquad (2)$$

we can write the characteristic damped oscillator differential equation:

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 = 0 \qquad (3)$$

Calculating the coefficients for a given pot was performed by matching the response of this idealized model to the aforementioned resonant frequencies and decays. Then the impedance boundary conditions for various surfaces within the cave could be estimated by creating an equivalent spatial distribution of the averaged impedances of the pots. Additionally, the relationship between the opening diameter of the pots and their absorption as well as their resonant frequency and absorption could be estimated and compared with the theoretical model.

4. FVTD SIMULATION

A second investigation was undertaken to integrate the previously calculated impedance boundary conditions into a FVTD simulation of the space. The model is detailed in [6], but we give an overview of the method, which produces an energy-stable simulation with arbitrary impedances on the boundaries.

Beginning with the linearized Navier Stokes equations, we notate:

$$\frac{1}{c^2}\partial_{tt}\Psi - \Delta\Psi = 0 \qquad (4)$$

where Ψ, the velocity potential of the field, is defined in terms of the particle velocity vector $\boldsymbol{v} = -\nabla\Psi$ and the sound pressure $p = \rho\partial_t\Psi$, where ∇ is the gradient operator, Δ the Laplacian operator, ρ the air density, c the speed of sound, and the first and second partial derivatives according to coordinate i are notated as ∂_i and ∂_{ii}, respectively. Gauss' theorem shows that any change in the energy stored in the acoustic field must be balanced with an equivalent change in energy at the domain boundaries.

At the boundary, we expect the energy stored and dissipated to depend on the pressure and normal velocity incident on a particular element of the surface. Bilbao et al. [6] utilize a parallel structure of

M one-port circuits to characterize the impedance relationship between the pressure and incident velocity, which is given as:

$$\hat{v}_{inc}(\boldsymbol{x}, s) = Y\hat{p}(\boldsymbol{x}, s) \tag{5}$$

in the Laplace domain (with s as the transform variable) and where the impedance Y is given by:

$$Y(\boldsymbol{x}, s) = \sum_{m=1}^{M} \frac{s}{L^m(\boldsymbol{x})s^2 + R^m(\boldsymbol{x})s + \frac{1}{C^m(\boldsymbol{x})}} \tag{6}$$

This is a convenient representation as the coefficients representing the pots in the Helmholtz resonator formalism may be directly translated into terms in the circuit model by means of electromechanical analogy.

The FVTD approach results in a two-step update equation for the velocity potential Ψ given by:

$$\delta_+\delta_-\Psi_j + \frac{c^2}{V_j}\sum_{k=1}^{N} \frac{\beta_{jk}S_{jk}}{h_{jk}}(\Psi_j - \Psi_k) + \frac{c^2}{V_j}\sum_{l=1}^{N_b} \gamma_{jl}S_l v_l = 0 \tag{7}$$

for a collection of N cells, each of which with volume V, intercellular distances to each of its neighbors h, and intercellular or cell-boundary surface areas S; where δ_+ and δ_- are forward and backward temporal difference operators, respectively, β and γ are indicator functions selecting only neighboring cells or boundaries, and v_l is the velocity incident upon a given boundary, computed from the local reactivity impedance boundary conditions. Our implementation of this scheme is available online [7].

4.1 Modeling and Meshing

From measurements, a wireframe model of the space was created in the 3D modeling package *Blender*. It was created from pictures in [1] and [3].

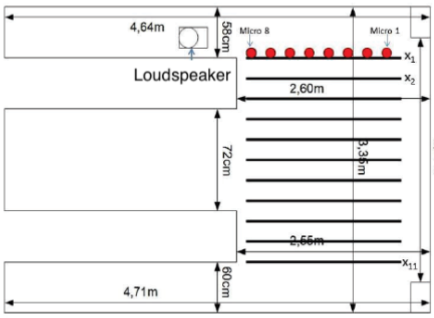

Figure 3 – Map with dimensions of the phonocamptic cave and positions of measurements performed in [3]

Because of the curvature of the cave, we decided to pursue an irregular meshing of the space in order to ensure excellent fit at the edges of the problem domain. Often, finite difference schemes use a cubic mesh for simplicity and computational efficiency, but such a meshing leads to error in the surface area modeled, which affects the resulting reverberation time. This “staircase effect” justifies the use of FVTD in modeling spaces with curved features. Ideally, we should use a hybrid meshing approach where most of the interior is modeled with regular elements and special elements are fit at the problem boundaries. But in the absence of specialized tools, a purely irregular meshing using tetrahedrons was undertaken using the *gmsh* package [8].

Irregular meshes also benefit from mitigation of dispersion error; however, in comparison to regular meshes, they can be challenging to parameterize and analyze.

4.2 Simulation Parameters

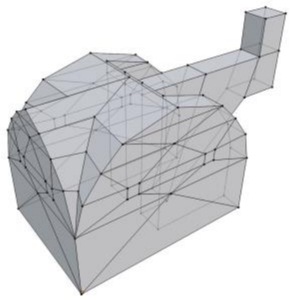

Figure 4 – Wireframe of cave geometry

For this study, the highest frequency simulated was 397 Hz. The mesh was ensured to only contain elements that satisfied the stability criteria for the FVTD scheme, where T_S is the sampling time interval:

$$\frac{1}{V_j}\sum_{k=1}^{N} \frac{\beta_{jk}c^2T_s^2S_{jk}}{2h_{jk}} \leq 1 \tag{8}$$

A spatial and temporal Gaussian was used as initial conditions. Its position is defined by the loudspeaker in Figure 3.

All walls were defined as either purely resistive, or using the model of acoustical pots. The resistive model was based on Sabine's formula, and the coefficients were computed according to the reverberation times with blocked pots [3]. The computed T_{60} was 0.58 seconds, the room volume was 28m^3, and the total surface area was 65m^2. The resulting absorption was applied to all of the non-pot surfaces.

As the highest frequency simulated was quite low, the meshed used in this study has been kept coarse, with a mesh size of approximately 40cm. Each zone of 12 pots was simplified into one square zone of side 36.47cm. Instead of 64 pots of 8.4cm of average diameter, the model has 6 square zones keeping the same total area, but with the equivalent impedance of the given group of pots.

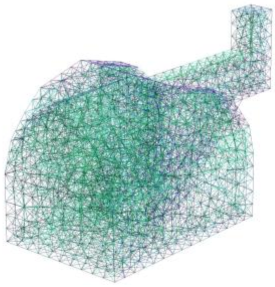

Figure 5 – Resulting nonuniform tetrahedral mesh

5. RESULTS

5.1 Acoustical Pots

After computing the RLC coefficients for each pot, they were compared with the physical dimensions and measured resonances to confirm the validity of the model.

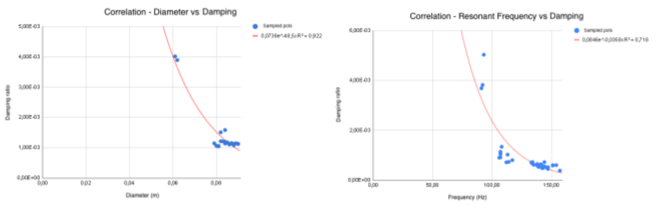

Figure 6 – Exponential relationship between: opening diameter and damping ratio (left); resonant frequency and damping ratio (right)

As can be seen, the expected exponential relationships were observed. This suggests that the model is well suited to modeling the pots and could be used in the acoustic simulation.

5.2 FVTD Simulation

Simulations were performed to compare with the known acoustical parameters of the space and better understand the impact of the tetrahedral meshing. The temporal evolution of a Gaussian impulse from the default loudspeaker position is shown in Figure 7.

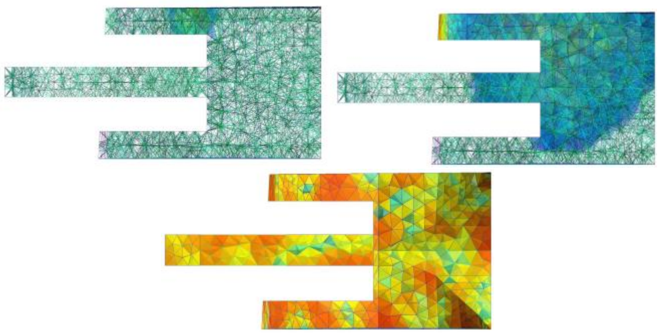

Figure 7 – Time evolution of simulated pressure field from initial impulse to diffuse field, viewed from above. Top left: after 1ms; top right: after 8ms; bottom: after 1.5s

One important observation is that all of the energy stored in the internal acoustic fields as well as in the boundary impedances (here, the capacitance of the pots) may be computed. It proved conservative to machine precision, confirming that the stability criterion was fulfilled.

The initial analysis of the simulations suggests that the overall room characteristics are matched. Further work is required to confirm the equivalent spatial distribution of pot impedances. But it is difficult to auralize the results yet. In the future, finer meshes and higher sample rates should make simulation of the wideband acoustics of the space possible.

6. CONCLUSIONS

In this paper, we have described an approach to modeling an acoustically and architecturally unique space with a combination of acoustic models for boundary conditions and acoustic wave motion. By using the model of a Helmholtz resonator to match physical pots embedded in the walls of the phonocamptic cave at the Cathedral of Noyon and computing the equivalent electroacoustic analogy impedance, the effect of these unique absorbers could be implemented in a wave-based acoustic simulation, well-suited for the expected modal effects in the range of frequencies under study.

7. REFERENCES

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