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Adapting the EST method to ancient theatres: a proposal.

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ABSTRACT

The paper investigates under which assumptions the EST method, initially developed for modelling the propagation of acoustical energy in flat spaces such as hallways and open space offices, can be adapted to unbounded spaces such as ancient theatres. It turns out that it mainly requires that the air column above any position in the open theatre contains finite acoustical energy, whatever its height. This is indeed the case since at high altitudes above the theatre, energy decreases with the square of the height due to the increasingly accurate assimilation of the theatre to a point source. In other words, one must use high enough elements, so that the intensity on the top of the elements can be considered as negligible, leading to negligible absorption and scattering on the top boundary. Therefore, one only needs considering absorption and scattering at the bottom boundary of the elements; and the integration on the elements must be revisited to account for the decrease of intensity with altitude. The corresponding bi-dimensional equations will be presented and solved for a variety of absorption and scattering coefficients on the surface of the theatre, and compared to measurements in an actual theatre. **Keywords:** energy-stress tensor, unbounded spaces, simulation.

1. INTRODUCTION

The energy-stress tensor (EST) formalism was introduced by Dujourdy *et al.* [1] to generalize the diffusion equation formalism of Ollendorf and Picaut [2, 3]. Indeed, where the diffusion equation arbitrarily introduces a gradient type relationship between sound intensity and total energy, the energy-stress tensor formalism introduces instead the conservation equation for intensity. When integrated in disproportionate enclosures, the stress-energy tensor yields both absorption and scattering on the boundaries. Dujourdy *et al.* showed that this formalism is able to simulate actual measurements in flat rooms, be they one-dimensional (hallways, [1]) or two-dimensional (open-space offices, [4]), which was later confirmed by Meacham *et al.* [5] in a hallway, taking into account source directivity. Common to all these simulations is the consideration that the vertical dimension is negligible, which is accounted for by integration on this vertical dimension while taking due consideration of the relevant boundary conditions, both on the floor and the ceiling.

The present paper intends to adapt the EST formalism to the opposite case of unbounded spaces, specifically to spaces without a ceiling, such as ancient Greek and Roman theatres. We shall show that it requires that the air column above any finite surface on the theatre contains finite sound power, an assumption that is obviously satisfied since any sound source emits finite sound power at any instant. After reviewing the background for the EST formalism, we shall derive the relevant equation by proper vertical integration, and solve the equation for a simplified theatre geometry with the code developed in [4]. Lastly, we shall compare the results with measurements in a Roman theatre.

2. BACKGROUND

2.1 The EST Formalism

Dujourdy *et al.* [1, 4] have shown that the wave equation, satisfied by the velocity potential, can be extended by a set of conservation equations that reduces to the conservation of the energy-stress tensor $\bar{\bar{T}}$:

$$\vec{\nabla}.\bar{\bar{T}} \qquad (1)$$

with

$$\bar{\bar{T}} = \begin{pmatrix} E_{tt} & E_{tx} & E_{ty} & E_{tz} \\ E_{tx} & E_{xx} & E_{xy} & E_{xz} \\ E_{ty} & E_{xy} & E_{yy} & E_{yz} \\ E_{tz} & E_{xz} & E_{yz} & E_{zz} \end{pmatrix} \qquad (2)$$

where $E_{tt} = E$ is the total energy, $(E_{tx}, E_{ty}, E_{tz}) = \vec{J}$ is the sound intensity vector, and where the remaining tensor is the wave-stress tensor introduced by Morse and Ingard [6]. Thus the two conservation equations

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are:

- the conservation of energy:

$$\frac{1}{c}\partial_t E + \vec{\nabla}.\vec{J} = 0 \qquad (3)$$

- the conservation of intensity:

$$\frac{1}{c}\partial_t \vec{J} + \vec{\nabla}.\bar{\bar{E}} = 0 \qquad (4)$$

Dimensional reduction was obtained by integrating eqs. 3 and 4 on the vertical dimension, respectively introducing the modified absorption coefficient *A* and the modified scattering coefficient *D* on both the floor and the ceiling. Dujourdy *et al.* [4] justify the form of these coefficients in terms of energy balance at the boundary, but the discussion goes beyond what is needed here. It should only be noted that the integration on the vertical dimension also introduces the mean free path that reduces to $\lambda = 2l_z$ for flat rooms, where l_z is the height of the room, leading to the system of equations:

$$\frac{1}{c}\partial_t E + \partial_x J_x + \partial_y J_y + \frac{\mathrm{A}}{\lambda}E = 0 \qquad (5)$$

$$\frac{1}{c}\partial_t \vec{J} + \frac{\mathrm{D}}{\lambda}\vec{J} + (\partial_x, \partial_y)\begin{pmatrix} E_{xx} & E_{xy} \\ E_{xy} & E_{yy} \end{pmatrix} = 0 \qquad (6)$$

Dujourdy *et al.* [4] further show that closing the system of equations requires to postulate that $E_{xx} = E_{yy} = E/2$ and $E_{xx} = 0$, amounting to isotropic distribution of energy in the horizontal directions. They then obtain a linear second-order hyperbolic equation, known as the Telegraph equation:

$$\frac{1}{c^2}\partial_{tt}E - \Delta\frac{E}{2} + \frac{\mathrm{A}+\mathrm{D}}{\lambda c}\partial_t E + \frac{\mathrm{AD}}{\lambda^2}E = 0 \qquad (7)$$

2.2 Dimensional Reduction in Unbounded Case

Dimensional reduction in the unbounded case is obtained in much the same way as in the flat case, except that energy densities are replaced by surface densities. Let us take the energy conservation. Integration of eq. 3 along the vertical axis *z* leads to:

$$\frac{1}{c}\int_z \partial_t E dz + \int_z \partial_x J_x dz + \int_z \partial_y J_y dz + \int_z \partial_z J_z dz = 0 \qquad (8)$$

Following the suggestion of [1], we relax the hypothesis that *E*, J_x and J_y are independent of the vertical coordinate, and consider instead the value of *E* taken at the bottom of the air column to define an equivalent "acoustical layer" of height *l*, so that total acoustical energy in the vertical column is equal to *El*. This makes it possible to define in turn an equivalent acoustical intensity, with takes the values J_x and J_y along the *x* and *y* axes, so that the vertical integrals of values J_x and J_y are equal to values $J_x l$ and $J_y l$. Taking further into account that J_z must be null on the top of the air column ($J_z^+ = 0$) because of the finite density assumption, the previous equation reduces to:

$$\frac{1}{c}\partial_t El + \partial_x J_x l + \partial_y J_y l - J_z^- = 0 \qquad (9)$$

where J_z^- is the vertical sound intensity into the floor. We then introduce the absorption coefficient α that links J_z^- to the energy density at the bottom of the air column:

$$-J_z^- = \alpha E \qquad (10)$$

and obtain for the conservation of energy in the whole air column:

$$\frac{1}{c}\partial_t E + \partial_x J_x + \partial_y J_y + \frac{\alpha}{l}E = 0 \qquad (11)$$

Similar integration leads to a new formulation of the conservation on sound intensity in the air column in the case of isotropic distribution of energy in the horizontal directions:

$$\frac{1}{c}\partial_t \vec{J} + \frac{\beta}{l}\vec{J} + \frac{1}{2}(\partial_x + \partial_y)E = 0 \qquad (12)$$

where β is a scattering coefficient. The telegraph equation then becomes:

$$\frac{1}{c^2}\partial_{tt}E - \Delta\frac{E}{2} + \frac{\alpha+\beta}{lc}\partial_t E + \frac{\alpha\beta}{l^2}E = 0 \qquad (13)$$

Dujourdy *et al.* [1] have shown that the steady state solution of eq. 13 becomes asymptotically proportional to $\exp\left(-\frac{\sqrt{2\alpha\beta}}{l}r\right)/\sqrt{t}$ for large values of *r*, the source-receiver distance.

3. MEASUREMENTS

As in [1, 4], the absorption and scattering coefficients are derived from measurements. But in the present case, we have one more unknown: the equivalent height of the acoustical layer. As the floor of ancient theatres is made of stones, we choose to fix its absorption coefficient, using values from the literature, and evaluate the equivalent height from reverberation time measurements.

3.1 The Roman Theatre of Carthage

In the summer 2014, we had the opportunity to carry out extensive measurements of the Roman Theatre in Carthage, Tunisia [7]. We therefore choose these measurements as references for simulations.

Figure 1 – The Roman Theatre of Carthage

The Roman Theatre of Carthage (Figure 1) was erected in the 2nd century AD and destroyed by the Vandals in the 5th century. Rediscovered at the end of the 19th century, it was subject to archaeological excavations in 1904-1905, and briefly again in 1967 prior to its reconstruction for the Festival of Carthage [8]. This reconstruction cannot be considered as faithful, as is evidenced by the vestiges that stick out of its cavea. It has a capacity of 10 000 spectators. Figure 1 presents a photo of the theatre taken during our measurement campaign. Note the superstructure on the left, remnant of the original theatre, which is used as sound booth during the Festival. And Figure 2 presents an approximate plan of the theatre including the measurement positions.

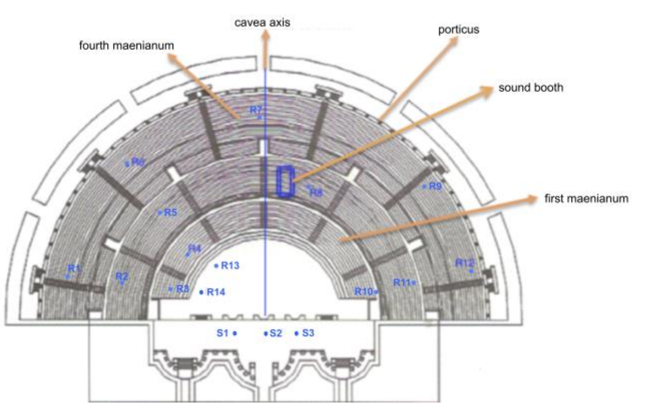

Figure 2 – Approximate plan of the Roman Theatre of Carthage. S: source positions; R: receiver positions

3.2 Reverberation Times

The mean reverberation times measured in the Roman Theatre are presented in Figure 3 for three aggregated bands corresponding to the low frequencies, the medium frequencies, and the high frequencies. The standard deviations show that they are relatively constant in the theatre. Consequently, computing average logarithmic decrements $c\alpha/l$ for the aggregated bands is meaningful: they are given in Table 1, where LF corresponds to the octaves 62, 125 and 250 Hz; MF to octaves 500 and 1k Hz; and HF to octaves 2k, 4k and 8k Hz.

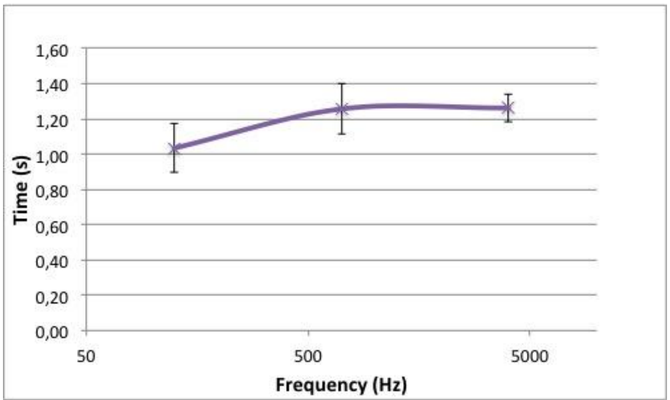

Figure 3 – Mean reverberation times and standard deviations in the Roman Theatre of Carthage

It turns therefore out that the height of the equivalent acoustical layer remains of the order of the meter, that is, it is consistent with the height of the listeners' ears above the ground level of the cavea tiers.

Table 1 – Mean logarithmic decrements, absorption coefficients and heights of "acoustical layer"

	LF	MF	HF
decrement	13.4	11.0	10.9
abs. coeff.	0.03	0.04	0.07
layer height	0.79	1.27	2.24

3.3 Spatial Decay

Spatial decays as measured in the Roman Theatre are presented in Figure 4 as functions of the source-receiver distance for the three aggregated bands: low frequencies, medium frequencies, and high frequencies.

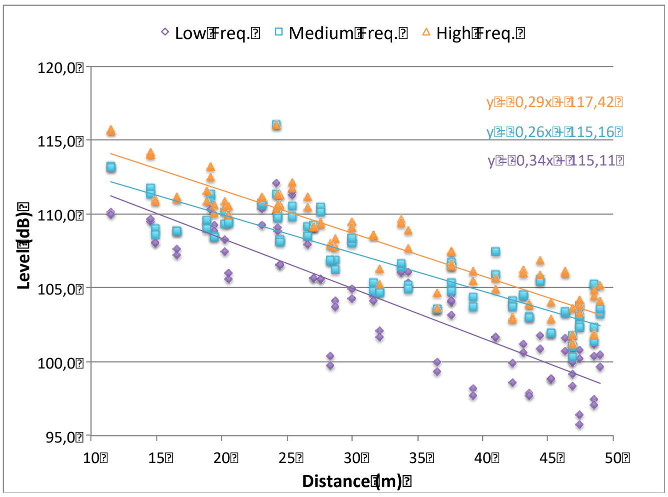

Figure 4 – Spatial decays in the Roman Theatre of Carthage

Short distance measurements in Figure 4 correspond to receiver positions located in the orchestra (R13 and R14), and do not behave differently than receiver positions in the cavea. Note the rather large dispersion of measured levels for similar source-receiver distances, probably due to air turbulence as is expected over long distances in open-air. Cicadas were also constantly singing at a recorder level of 30 dB, and probably interfered with the measurements at the furthest receiver positions. As levels were monitored at on fixed position in the orchestra, it should be possible to reduce dispersion; but this has not been further investigated at the present stage.

For distances above 20 m, the data of Figure 4 indicate a spatial decay for all three bands consistent with the asymptotic expression of spatial decay in the EST model. From the regression lines, we can estimate the scattering coefficients for the three bands: they are given in Table 2, which shows that scattering coefficients are only slightly larger than absorption coefficients, but for high frequencies. According to [1, 4], it indicates that accurate evaluation of the absorption and scattering coefficients need iterations to obtain exact values.

Table 2 – Mean logarithmic decrements and scattering coefficients for spatial decays

	LF	MF	HF
decrement	0.07	0.06	0.08
scatt. coeff.	0.05	0.07	0.22

It should be noted, however, that investigations of a few impulse responses selected randomly showed that they are dominated by the direct sound and a few strong reflections - see Figure 5 for an example corresponding to S2R5.

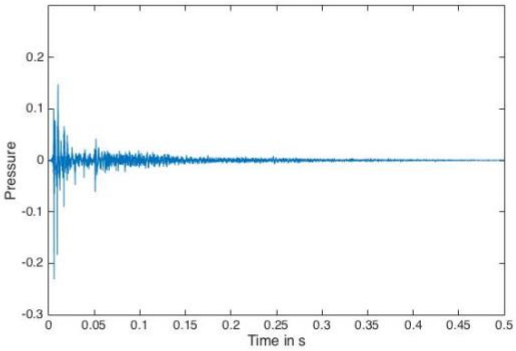

Figure 5 – Typical impulse response measure in the Roman Theatre of Carthage

4. DISCUSSION

Our measurement data in the Roman Theatre of Carthage are consistent with the spatial decays of the energy-stress tensor model that generalizes the diffusion model of Ollendorf and Picaut [2, 3]. However, preliminary investigations, using raw measurement data that were not properly compensated for the changing setting-ups of the measuring equipment, did not show this consistency. As a consequence, we did not carry out simulations of the Roman Theatre of Cartage with our EST code. Nevertheless, the impulse responses are dominated by the direct sound and two or three strong reflections. According to Canac [9], these reflections are created on lower tiers on the opposite side of the cavea. But apparent on Figure 5 is a residual reverberation field below these strong components. It suggests that removing the direct sound and the strong reflections in order to recover the diffuse reverberant field could improve the consistency with the EST model. Indeed, the EST model is only valid for diffuse reverberant fields. Removing strong reflections could be achieved using Matching Pursuit as in [10]. Another technique is manual removal of the reflections, and filling the gap with data derived from signals before and after the gap, as proposed by [11] for removing clicks in old gramophone recordings. None of these methods has been tested so far.

5. CONCLUSIONS

The energy-stress tensor method, in short EST method, was developed to efficiently simulate the diffuse reverberant field of an enclosed space. Despite the fact that the impulse responses of an open space are dominated by the direct sound and a few strong reflections coming from opposite lower sections of the cavea, there exists a diffuse reverberant field, as illustrated in Figure 5, that is sufficient to make spatial decays in open-air theatres consistent with the EST model. There remains to check whether the EST method can predict this reverberant field, and whether a hybrid simulation procedure, similar to Meacham *et al.*'s hybrid procedure for hallways [12] can be applied to open-air theatres.

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