Theoretical investigation of diffraction phenomena in the ancient theatre of Epidaurus

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ABSTRACT

The effect of edge diffraction in ancient theatres is investigated. A cross section of the theater of Epidaurus simplified as right-angled steps is considered. A recently presented solution for computing the impulse response around a rigid wedge is employed for the theoretical study of diffraction by the edges of the steps. It is shown that diffracted signals coming from edges below a listener come close together and with relative small amplitude. Diffracted signals from edges above the listener come further apart, some with very large amplitude and almost all with negative polarity. A new parameter is proposed that predicts the effect of edge diffraction based solely on geometrical characteristics. It is shown that the height of the source and the inclination of the steps affect the diffraction contributions the most. The effect of the various diffraction contributions to acoustic indices is considered next. For the computation of the acoustic indices the relative strength between geometrical and diffraction contributions, which are different in nature, must be determined. In the present work this is done via the unit step responses. Upper diffractions affect negatively the acoustic indices, considerably more than lower diffractions.A source positioned at the height of a deus ex machina improves the acoustic indices compared to a source at the height of a standing actor.Large inclination angles of the steps affect negatively the acoustic indices. Finally, results including the effect of all geometrical and all diffraction contributions are compared with published measured data for the theater of Epidaurus.

Keywords: Edge diffraction

1. INTRODUCTION

The effect of sound diffraction on the acoustics of ancient theatres is investigated. Consider a cross section of the ancient theatre, a point source at the centre of the orchestra and a listener seated at an arbitrary seat row as shown in Fig.1. Sound emitted from the source reflects on the orchestra and on the surfaces of the steps, diffracts on the edges of the steps or follows a combination of reflections and diffractions until it reaches the listener. The subject of the current work is the diffraction caused by the edges of the steps.

The aim of the present work is not to present a new prediction method for the acoustic field. Many commercial, as well as in-house predictions tools are available (for example ODEON, CATT, OliveTree Lab). The present work focuses on analyzing the diffraction phenomenon and the signals it causes. In the first part of the work the impulse responses of the diffracted contributions are analyzed, while in the second part the effect of diffraction on acoustics indices is investigated. For the first two parts of the work only the incident signal and the diffracted signals caused by it are considered, while the geometry is a simplified step geometry, i.e. without the diazoma separating the lower from the upper koilon, and without taking into account the different inclination angles in the upper and lower koilon (see Fig. 1). Finally, in the last section, all geometrical (direct signal/reflections) and al diffraction contributions are considered for the cross section of the theatre or Epidaurus (including the diazoma and the different inclination angles).

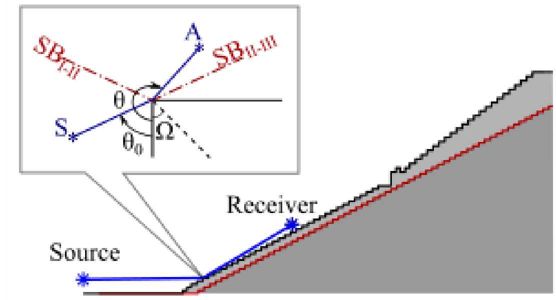

Figure 1 – Cross section of the theatre of Epidaurus (black line), of a simplified step geometry (red line). A source-wedge-receiver configuration is demonstrated in the inlet.

A previous work by Farnetani et. al. [1] also investigated the diffracted signals by employing a different edge solution. The primary concern was the application and validation of the edge solution for configurations of ancient theatres. The present work goes further into the investigation of diffracted signals, presenting new parameters that can describe diffraction based

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solely on geometrical characteristics, and identifying the geometrical parameters that play an important role. Furthermore, unlike the previous work, the effect on acoustic indices is investigated in the present work. For this to be accomplished, the total impulse response must be considered and the relative strength of diffracted and geometrical acoustics contributions must be determined.

2. DIFFRACTION CONTRIBUTIONS

Consider a wedge of angle $2\Omega = 90^{\circ}$, a point source S and a receiver A around the wedge. For the theoretical study of diffraction an approximate solution is considered that provides the impulse response at A [2]

$$p_{irf}(A) = -\frac{1}{4\pi}\frac{1}{\sqrt{rr_0}}\frac{2}{\sqrt{t^2-t_d^2}}\sqrt{2}\gamma\sqrt{\frac{t}{t_d}+1}\cdot\gamma\bar{t}\Phi_{\pm}\cdot$$

$$\cdot\left(1/\left(2\left(t-t_d\right)\left(\bar{t}\gamma^2-\cot\left(\frac{\pi}{\gamma}\right)\gamma\bar{t}\Phi_{\pm}\right)+(\gamma\bar{t})^2\Phi_{\pm}^2\right)\right), \quad (1)$$

$$\Phi_{\pm} = \gamma\left(\cos\left(\frac{\theta\pm\theta_0}{\gamma}\right)-\cos\left(\frac{\pi}{\gamma}\right)\right)/\sin\left(\frac{\pi}{\gamma}\right), \bar{t} = \frac{rr_0}{Lc}$$

where (r_0, θ_0) and (r, θ) are the coordinates of source and receiver respectively, (see Fig. 1), Ω is the half angle of the wedge, $L = r_0 + r$ the path the sound travels from the source to the receiver after undergoing diffraction on the edge of the wedge, and $t_d = L / c$ *(arrival time)* the time the diffracted signal arrives at the receiver. Finally, it is noted that for wedges with angle $2\Omega = 135^{\circ}$ the diffracted signal is identically zero. As a result, in our geometry only the wedges with $2\Omega = 90^{\circ}$ produce diffracted signals.

The area around the wedge is separated into three regions by two shadow boundaries SB_{I-II} and SB_{II-III} (see dashed lines in Fig.1). The diffracted signal obtains its largest values close to the shadow boundaries. Also, at the shadow boundaries the diffracted signal undergoes a change of polarity (a positive signal on one side of the shadow boundary becomes negative).

Consider the cross section of the simplified step geometry, a point source at the centre of the orchestra 1.5m above the ground and a listener seated at an arbitrary seat row0.2mfrom the outer edge of the seat and 0.8mabove the seat. Consider that there are no reflections on the surfaces of the theatre. The impulse response at the listener is the summation of the incident signal coming directly from the source and of all the diffracted signals coming from each one of the diffracting edges. Figure 2 shows the diffracted signals from all edges at the listener location. Blue coloured pulses correspond to diffracted signals coming from edges lower than the listener (*lower diffracted signals*), while red coloured pulses correspond to diffracted signals coming from edges higher than the listener (*upper diffracted signals*).

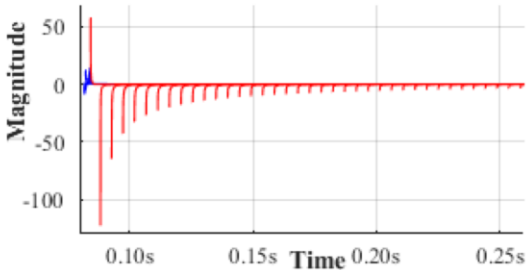

Figure 2 – Diffracted signals arriving at a listener at the 20th step.

3. CHARACTERISTICS OF DIFFRACTED SIGNALS

In this section the characteristics of the diffracted signals are discussed. Lower diffracted signals arrive close together and with relatively small amplitude. Upper diffracted signals arrive further apart, some with very large amplitude and almost all with negative polarity (see Fig.2).

Figure 3(left) shows the arrival times of the diffracted signals. For edges that are higher than the listener the arrival times increase significantly the further above the orchestra the diffracted edge is located (red marks). For edges that are lower than the listener the arrival times remain roughly the same (blue marks). Accordingly, the lower diffracted signals come close in time, while upper diffracted signals come further apart,(being almost resolved in time). Furthermore, the arrival time for some of the lower diffractions seems to be the same with some of the upper diffractions. This explains the partial overlapping of lower and upper diffracted signals in Fig. 2. A final observation can also be made. For the upper diffracted signals, the arrival rime increases linearly. As e result, the first upper signal comes from the edge that is immediately above the listener. The corresponding change for lower diffracted signal is not linear. As a result, the first lower diffracted signal comes from a few steps below the listener.

Figure 3(right) depicts a *polarity plot*, derived directly from Eq. (1) for $\Omega = 45^{\circ}$, that shows the polarity of the diffracted signal as a function of the angular position of source and receiver for each diffraction problem. The red marks correspond to upper diffracted signals, the blue marks to lower diffracted signals. Marks in the yellow areas indicate signals of negative polarity. Marks in the green areas indicate positive diffracted signals. The straight diagonal lines correspond to the shadow boundaries. Marks close to the shadow boundaries indicate diffracted signals with large amplitude. Marks close to the curved line indicate signals with small pulse amplitude.

Finally, it is emphasized that the time evolution of the diffracted signals is quite different from that of geometrical acoustics contributions. The latter are described mathematically as Dirac functions, while the former have an infinitely large amplitude at the arrival time and a long lasting time decay.

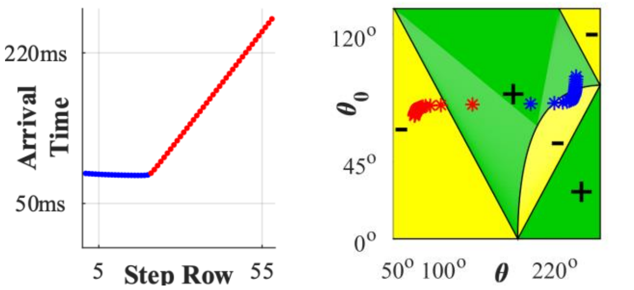

Figure 3 – Time arrivals (left) and polarity (right) of the diffracted signals shown in Fig.2

4. A NEW PREDICTION QUANTITY

Based on Eq. (1) a new quantity is proposed to predict the amplitude and polarity of the diffracted signals based solely on geometrical parameters:

$$M = -\left(\Phi_+^{-1} + \Phi_-^{-1}\right) / \sqrt{r \cdot r_0} \qquad (2)$$

The parameter M is computed for the case considered in Fig.2. The parameter M for each diffracting edge (each step) is presented versus the arrival time that corresponds to such step (see Fig.3). The combined parameter $M(t_d)$ (shown in Fig.4) correlates very well with the diffracted signals shown in Fig.2.

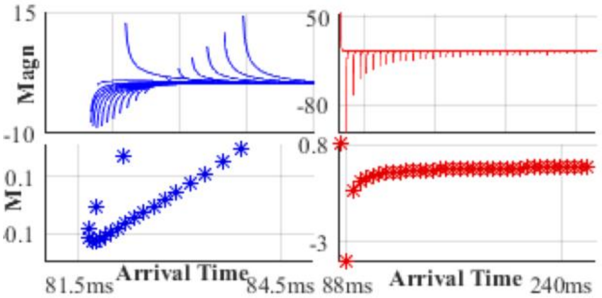

Figure 4 – The parameter M correlates well with the lower and upper diffractions shown in Fig 2.

5. EFFECT OF GEOMETRICAL PARAMETERS

The quantity $M(t_d)$ shows that the effect of diffraction can be almost completely analyzed based on the geometrical characteristics. In this section, changes in the geometry of the source-edge-receiver configuration are considered.

Changing the step on which the listener is seated, does not change the characteristics of the diffracted signals.

Changes in the location of the receiver on a given step (for example, moving the receiver closer or away from the back of the seat) do not affect the diffracted signals.

Changes in the height of the source can affect the diffracted signals considerably. Figure 5 shows the diffracted signals that arrive at a listener at the 20th step for two different source heights: 7.5m and 15m. As the source height increases, the lower diffracted signals arrive more resolved in time, they overlap more with the upper diffracted signals and they obtain larger amplitudes. Indeed, the arrival times for the different source heights are shown in Fig. 6(left), where the effect, particularly on the lower diffracted signals, can be observed. The larger amplitude of the lower diffracted signals shown in Fig. 5 compared to those shown in Fig. 2 is attributed to the different location of the shadow boundary as the source height increases. Figure 6(right) shows the shadow boundaries (dashed lines) of the diffraction contribution originating from the 8th step that reaches the listener at the 20th step. As the source height increases, the shadow boundary SB_{II-III} at the 8th step changes its angular location. The receiver at the 20th step is closer to the shadow boundary. Accordingly, the diffracted pulse has larger amplitude.

Changes in the inclination of the steps can also affect the diffracted signal. The higher the inclination angle, the later the diffraction contributions arrive.

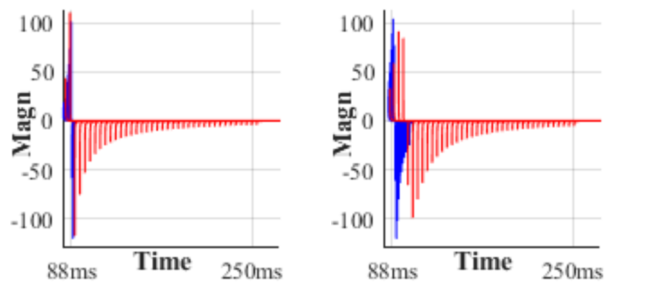

Figure 5 – Diffracted signals arriving at a licenser at the 20th step for two different source heights: 7.5m (left) and 15m (right) above the orchestra.

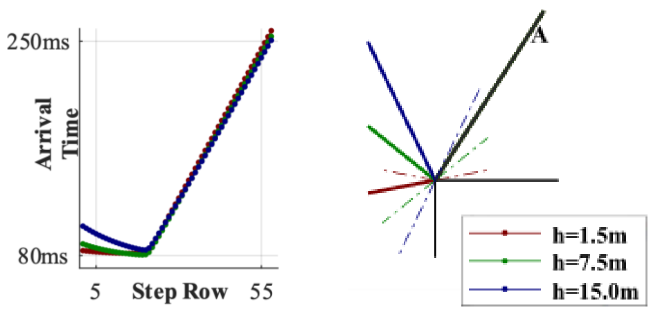

Figure 6 – (left): Time arrivals of the diffracted signals at a listener at the 20th step for three different source heights; (right): shadow boundaries of the diffraction contribution at the same listener originating from the 8thstep.

6. EFFECT ON ACOUSTICS INDICES

In the present section the effect of diffraction is expressed in terms of acoustic parameters/indices. Specifically, the echo criterion (TS)

$$TS_{speech} = \left(\int_0^{\tau} t \cdot |p(t)|^{2/3} \, dt \right) / \left(\int_0^{\tau} |p(t)|^{2/3} \, dt \right) \qquad (3)$$

is considered, where $p(t)$ is the impulse response at the listener and smaller values of TS are acoustically preferable. In the following, improvement of the acoustic index will mean decrease of its value. It is emphasized that the values of TS presented are a mere indication of the general trend.

An important observation before presenting the results is merited. Because of the different nature of geometrical acoustics contributions and diffraction contributions, their relative strength in the total impulse response cannot be determined. The corresponding unit step responses must be used instead. The diffraction solution employed here [Eq. (1)], unlike other solutions, is integrable with time and the unit step response can be obtained analytically.

Figure 7 shows the relative importance of the various contributions (incident signal, lower diffractions, and upper diffractions) expressed in terms of TS for all steps. It can be observed that diffraction contributions deteriorate the acoustic index: the lower diffractions by little, the upper diffractions considerably. Also, diffraction seems to homogenize the acoustic index among the different steps.

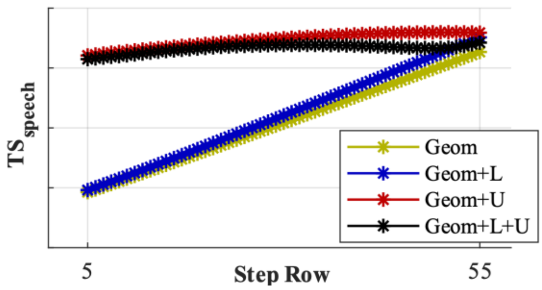

Figure 7 – The effect of lower and upper diffractions in the value of TS at all steps.

Figure 8 shows the effect of the source height and of the inclination angle (discussed in the previous section) on TS. Compare two source heights [Fig.8(left)]:1.5m corresponding to a standing actor (red) and 7.5 m corresponding roughly to a deus ex machine (green). The source at the deus ex machine height offers a better acoustic index. On the right side of Figure 8, the effect of the inclination angle is presented. Three inclination angles are considered, 24°, 30° and 45°(creating cross sections that roughly correspond to the theater of Epidaurus (black), of Aosta (red) and of a Mayan theater (green), respectively).The acoustic index is better for small inclination angles and worse for large inclination angles.

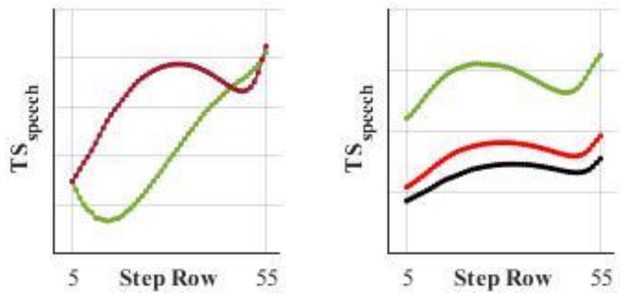

Figure 8 – Effect of source height (left) and inclination angle (right) in the value of TS at all steps.

7. SOUND FIELD AND ACOUSTIC PARAMETERS

The preceding analysis describes the diffraction contributions originating from the incident signal. Similarly, sound after reflecting on the orchestra reaches the edges of each step and produces a similar pattern of diffracted signals. Geometrical acoustic contributions coming after reflections on the seats and on the back of the seats also produce diffracted signals. Figure 9 shows all geometrical contributions and all diffracted signals generated by each one of the geometrical acoustics contributions for listeners at step 5 and step 29 in the theatre of Epidaurus (see Fig. 1). In the inlet the comparison of the clarity index C_{80} computed by in-situ measurements (taken from Ref. [3]) and by the analytical impulse responses is reported.

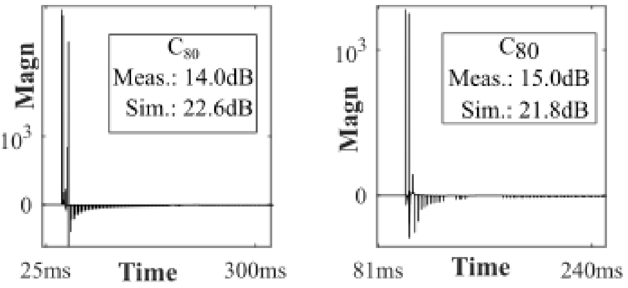

Figure 9 – Impulse response containing all geometrical and diffraction contributions at step 5 (left) and 29 (right) of the theatre in Epidaurus.

8. SUMMARY

The effect of edge diffraction in ancient theatres is investigated. The diffraction contributions depend mainly on geometrical characteristics: the length between source-edge-listener, and the proximity of the listener to the shadow boundary. A new quantity is derived to describe this dependence. Regarding the diffracted signals: diffracted signals coming from edges below a listener (lower diffractions) come close together and with relative small amplitude; diffracted signals from edges above the listener (upper diffractions) come further apart, some with very large amplitude and almost all with negative polarity. Regarding the acoustic indices: upper diffractions affect negatively the acoustic indices, considerably more than lower diffractions; a source positioned at the height of a deus ex machina improves the acoustic indices compared to a source at the height of a standing actor; and large inclination angles of the steps affect negatively the acoustic indices.

9. REFERENCES

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